CHANDIGARH UNIVERSITY

Discover. Learn. Empower.

INSTITUTE - UIE

Bachelor of Engineering (Computer Science & Engineering)

Subject Name : CALCULUS & VECTOR SPACES

Subject Code : 20SMT-175

BE :CSE(All IT branches)

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Introduction to Linear Transformation

Course Outcomes

CO Number	Title	Level
CO1	1) The concept of partial derivatives and its application in real life situations 2) The concept of Multiple Integrals and its applications.	**Remember & Understand**
CO2	The concept of Group theory and its application of analysis to Engineering problems.	**Remember & Understand**
CO3	The concept of vector spaces in a comprehensive manner.	**Remember & Understand**

- Course prerequisites

- Basic Knowledge of Sets.
- Basic Knowledge of **binary operations**.
- Basic Knowledge of functions.
- Basic Knowledge of Matrices.

Topic Outcomes

- Students will able to understand basic concept of linear transformation.

- Students will able to understand basic concept kernel and nullity.

- Students now able to aware with the algebraic operations between linear transformations.

Linear Transformation

Let $f: V \longrightarrow F$ where V is vector space over the field F such that

(i) $T(x+y) = T(x) + T(y)$ for all $x, y \in V$

(ii) $T(\alpha x) = \alpha T(x)$ for all $x \in V,\ \alpha \in F$

The two above conditions can be written as

$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$ for all $x, y \in V, \alpha, \beta \in F$

Examples:

1) T(x)=0 $\forall x$ (zero transformation)
2) T(x)=x $\forall x$ (Identity Linear transformation)

Linear Transformation

Results:

- Let V be a vector space over F. If $(\alpha_1, \alpha_2, \dots, \alpha_n)$ is basis of V and W contains with vectors $(\beta_1, \beta_2, \dots, \beta_m)$ for $m \geq n$. Then exists unique linear tranformation $T: V \longrightarrow W$ such that $T(\alpha_i) = \beta_j, j = 1,2, \dots, n$.
- Let $T: V \longrightarrow W$ be Linear Transformation. Given vectors $v_1, v_2, \dots, v_n \in V$

(i) If $v_1, v_2, \dots, v_n$ are linearly dependent, then $\mathrm{T}(v_1), T(v_2), \dots, T(v_n)$ linearly dependent.

(ii) $\mathrm{T}(v_1), T(v_2), \dots, T(v_n)$ linearly independent, then $v_1, v_2, \dots, v_n$ are linearly independent

Linear Functional

- Let V be a vector space over a field F. A mapping $f: V \to F$ is termed a linear functional if for every $u, v \in V$ and every $a, b \in F$
$$f(au + bv) = af(u) + bf(v)$$

Examples:

1. Let $f: F^n \longrightarrow F$ be the ith projection mapping $f_i(a_1, a_2, \dots, a_n) = a_i$ then f_i is linear and so it is a linear functional on F^n.
2. Let V be the vector space of polynomials in t over R, Let $\phi: V \longrightarrow R$ be the integral operator defined by $\phi\big(p(t)\big) = \int_0^1 p(t)dt$. Since $\phi$is linear and hence it is a linear functional on V.

Kernel and Range

- **Kernel:**

Kernel of homomorphism T contains all those members that are mapped to 0 and it is called the Null space of T.

$Ker\,T = \{0\}$ iff T is one-one.

- **Range:**

Range of T is defined to be $\{T(x) | x \in V\} = R_T$

Rank and Nullity

- **Rank and Nullity of T:**

Let $T: V \longrightarrow W$ be a L.T. we define Rank of T= $\dim Range\, T = r(T)$

Nullity of $T = \dim KerT = \nu(\mathrm{T})$

- **Sylvester's Law :**

Let $T: V \longrightarrow W$ be a L.T., then $Rank(T) + Nullity(T) = \dim V$.

Algebra of linear transformations

Linear transformations can be added, and multiplied by scalars. Hence they form a vector space themselves.

- **Theorem:** Let T,U:V⟶W linear transformation if

  1) We define T+U:V ⟶ W by (T+U)(a)=T(a)+U(a).

  2) We define cT:V ⟶ W by cT(a)=c(T(a)).

  Then they are linear transformations.

- **Hom(V,W):**

Set of all L.T from vector space V to W over F. It is denoted by L(V,W) of Hom(V,W)

- L(V,W) is a vector space.

Algebra of linear transformations

Results:

- Let V and W be vector spaces of dimensions m and n over F. Then

$$\dim L(V,W) = m.n$$

- If $T_1, T_2 \in Hom(V,W)$ then

(i) $r(\alpha T_1) = r(T_1) \,\forall\, \alpha (\neq 0) \in F$

(ii) $|r(T_1) - r(T_2)| \leq r(T_1 + T_2) \leq r(T_1) + r(T_2)$

Invertible and Non Singular transformations

Definition:

$T: V \to W$ is invertible if there exists U: $W \to V$ is such that $UT = Iv$ and $TU = I_w$. Further, U is denoted by T^{-1}.

Result: If T is linear, then T^{-1} is linear.

Definition:

$T: V \to W$ is nonsingular if $T(c) = 0$ implies $c = 0$

Result:

T is nonsingular iff T carries each linearly independent set to a linearly independent set.

References

Reference Books

- Elements of Discrete Mathematics, (Second Edition) C. L. Liu, McGraw Hill, New Delhi, 2017
- Graph Theory with Applications, J. A. Bondy and U. S. R. Murty, Macmillan Press, London.
- Topics in Algebra, I. N. Herstein, John Wiley and Sons. Digital Logic & Computer Design, M. Morris Mano, Pearson.

Online Video Sites:

1. NPTEL
2. Coursera
3. Unacademy

THANK YOU